M.A./M.Sc. Semester –I Paper -1

MAT1TH01: GROUP THEORY

Unit 1: Isomorphism theorems for groups, Symmetric groups, Alternating groups, Dihedral groups, Matrix groups, Isometry groups of R^2 and R^3, Internal and External direct product and their relationship, Indecomposable groups.

Unit 2: Subnormal and normal series, Zassenhaus' lemma, Schreier's refinement theorem, Composition series, Jordan-Hölder's theorem, Chain conditions.

Unit 3: Action of a group G on a set, Stabilizer subgroups and Orbit decomposition, Class equation of an action, Burnside's theorem, Transitive and effective actions, Equivalence of actions, core of a subgroup.

Unit 4: Sylow subgroups, Sylow's Theorem I, II and III, p-groups, Examples and applications, Groups of order pq, Direct and inverse images of Sylow subgroups.

Unit 5: Commutator subgroup and commutator series of a group, Solvable groups, Solvability of subgroups and factor groups and of finite p-groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

Books Recommended:

1. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. J. A. Gallian, Contemporary Abstract Algebra, 4th Edition, Narosa Publ. House, 1998.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. J. B. Fraleigh, A first Course in Abstract Algebra, Pearson Edu. Inc., 2002.
3. Ramji Lal, Algebra, Vols. I & II, Shail Publications, Allahabad, 2002.

M.A./M.Sc. Semester –I Paper -2

MAT1TH03: COMPLEX ANALYSIS

UNIT ONE: Complex differentiability, Cauchy-Riemann equations, analytic functions, harmonic functions, harmonic conjugates, analyticity of functions defined by power series, the exponential function and its properties.

UNIT TWO: Branch of logarithm, power of a complex number, basic properties of contour integration, M-L inequality, fundamental theorem of contour integration, Cauchy's integral theorem, Cauchy-Goursat theorem (statement only), Cauchy's integral formula, Cauchy's integral formula for higher derivatives (statement only), Morera's theorem.

UNIT THREE: Maximum modulus theorem, Schwarz lemma, Taylor's theorem, Cauchy's estimate, zeros of an analytic functions, the identity theorem for analytic functions, Liouville's theorem, the fundamental theorem of algebra, Laurent's theorem.

UNIT FOUR: Singularities of functions, removable singularity, poles and essential singularities, Casoratti-Weierstrass theorem, residues, Cauchy's residue theorem, evaluation of simple definite integrals using contour integration, meromorphic functions, argument principle, Rouche's theorem, open mapping theorem, singularity and residue at ∞.

UNIT FIVE: Conformality, Möbius transformations, the group of Möbius transformations, cross ratio, invariance of circles, determination of Möbius transformations mapping real line onto itself, upper half plane onto itself, upper half plane onto open disc and an open disc onto an open disc.

Books Recommended:

1. J. B. Conway, Functions of One Complex Variable, Narosa Publ. House, New Delhi, 2002.
2. S. Ponnusamy and H. Silverman, Complex Variables, Birkhäuser, Inc., Boston, MA, 2006.
3. J. Bak, Complex Analysis, Springer, 1996.
4. V. Ahlfors, Complex Analysis (Third Edition), McGraw-Hill, 1979.
5. A. R. Shastri, An Introduction to Complex Analysis, Macmillan India Ltd., 1999.

M.A./M.Sc. Semester –I Paper -3

MAT1TH05: POINT-SET TOPOLOGY

Unit 1: Topological Spaces, metric topology, ordered topology, open sets, closed sets; interior, exterior, boundary and closure of a set, limit points of a set, characterization of closed sets and dense sets, separable spaces,basis and sub-basis of a topology, first countable and second countable spaces.

Unit 2: Sequences in a metric space, convergence of a sequence, complete metric spaces, nets and filters, continuous maps and their characterization, open maps, closed maps, homeomorphisms, topological invariants.

Unit 3: Product topology, Quotient topology and identification spaces (torus, projective spaces $P^{n,}$ Moebius strip and Klein bottle), connected spaces, locally connected spaces, path connected and locally path connected spaces.

Unit 4: Separation Axioms: T_0 spaces, T_1 spaces, T_2 spaces, regular spaces, T_3 spaces, completely regular spaces, normal spaces, Tychonoff spaces, T_4 spaces, characterization of these spaces, Urysohn's lemma, Tietze's extension theorem, Urysohn's embedding and metrization theorem.

Unit 5: Compact spaces and their characterizations, compactness in metric spaces and their characterisation (limit point compactness, sequential compactness, complete and total boundesness), locally compact spaces, Tychonoff's theorem, one point compactification.

Books Recommended:

1. J. L. Kelley, General Topology, Van Nostrand, 1995.
2. K. D. Joshi, Introduction to General Topology, Wiley Eastern, 1983.
3. James R. Munkres, Topology, 2nd Edition, Pearson International, 2000.
4. J. Dugundji, Topology, Prentice-Hall of India, 1966.
5. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
6. S. Willard, General Topology, Addison-Wesley, 1970.

M.A./M.Sc. Semester –I Paper -4

MAT1TH07: DIFFERENTIAL GEOMETRY I

UNIT ONE: Curves in space R^3, parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet- Serret formulae, Frenet approximation of a space curve.

UNIT TWO: Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of R^3,, fundamental theorem of space curves, surfaces in R^3,, regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on R^3,, surfaces of revolution, tangent vectors, tangent plane, differential of a map.

UNIT THREE: Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

UNIT FOUR: Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

UNIT FIVE: Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on asurface of evolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).

Books Recommended:

1. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
2. B. O' Neill, Elementary Differential Geometry, Academic Press, 1997.
3. A. Gray, Differential Geometry of Curves and Surfaces, CRC Press, 1998.
4. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.
5. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer, 1979.
6. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
7. L. P. Eisenhart, A Treatise on the Differential Geometry of Curves and Surfaces, Ginn and Company, Boston, 1909.

M.A./M.Sc. Semester –I Paper -5

MAT1TH09: CLASSICAL MECHANICS

UNIT ONE: The momentum of a system of particles, the linear and the angular momentum, rate of change of momentum and the equations of motion for a system of particles, principles of linear and angular momentum, motion of the centre of mass of a system, theorems on the rate of change of angular momentum about different points, with special reference to the centre of mass, the kinetic energy of a system of particles in terms of the motion relative to the centre of mass of the system.
Rigid bodies as systems of particles, general displacement of a rigid body, the displacement of a rigid body about one of its points and the concept of angular velocity, computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately, kinematical examples.

UNIT TWO: The angular momentum and the kinetic energy of a rigid body in terms of inertia constants, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes.
Euler’s dynamical equations of motion, motion under no forces, the invariable line and the invariable cone, Eulerian angles and the geometrical equations of Euler.

UNIT THREE: Generalized co-ordinates, Geometrical equations, holonomic and non-holonomic systems, Configuration space, Lagrange’s equations using D’ Alembert’s Principle for a holonomicconservative system, deduction of equation of energy when the geometrical equations donot contain time t explicitly, Lagrange’s multipliers case.

UNIT FOUR: Deduction of Euler’s dynamical equations from Lagrange’s equations, Theory of small oscillations, Lagrange’s method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, Lagrange equations for impulsive motion.

UNIT FIVE: Generalized momentum and the Hamiltonian for a dynamical system, Hamilton’scanonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, Phase space and Hamilton’s variational principle, the principle of least action, Canonical transformations, Poisson-Brackets, Poisson-Jacobi identity, Hamilton-Jacobi theory (outline only).

Books Recommended:

1. E. A. Milne, Vectorial Mechanics, Methuen & Co. Ltd., London, 1965.
2. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.
3. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
4. N. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.

M.A./M.Sc. Semester –III Paper -1

MAT3TH01: FIELDS AND GALOIS THEORY

Unit 1: Eisenstein's irreducibility criterion, Characteristic of a field, Prime subfields, Field extensions, Finite extensions, Simple extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields.

Unit 2: Splitting fields and their uniqueness. Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability.

Unit 3:Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, normal closures, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Unit 4:Primitive element theorem, Finite fields, Existence and uniqueness, Subfields of finite fields, Characterization of cyclic Galois groups of finite extensions of finite fields, fundamental theorem of algebra.

Unit 5: Cyclotomic extensions and polynomials, cyclic extensions, Solvability by radicals, Galois' characterization of such solvability, Generic polynomials, Abel-Ruffini theorem, geometrical constructions.

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y.,2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. T. W. Hungerford, Algebra, Springer (India), Pvt. Ltd., 2004.

M.A./M.Sc. Semester –III Paper -2

MAT3TH03: FUNCTIONAL ANALYSIS

UNIT ONE: Normed linear spaces, examples and its topological properties, Banach spaces, continuous linear transformations, spaces of continuous linear transformations from a linear space to a Banach space, continuous linear functionals.

UNIT TWO: Hahn-Banach theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem (or the Uniform boundedness principle).

UNIT THREE: Conjugate spaces, natural embedding of N in N^{**}, weak and weak*-topology on a conjugate space, conjugate of an operator, simple applications to reflexive separable spaces.

UNIT FOUR: Hilbert Spaces, Schwarz's inequality, orthogonal complement, Bessel's inequality, orthonormal sets, continuous linear functionals on Hilbert spaces, Riesz representation theorem, reflexivity of Hilbert Spaces, adjoint of an operator on a Hilbert space.

UNIT FIVE: Self-adjoint and normal operators, unitary operators on a Hilbert space, projections on Hilbert spaces, determinant and the spectrum of an operator, spectral theorem.

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.

M.A./M.Sc. Semester –III Paper -3

MAT3TH05: THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

UNIT ONE:

Initial and Boundary Value Problems, Picard's Iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Examples of Lipschitzian and Non-Lipschitzian functions, Picard's Theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, examples of problems without solutions and of equations where Picard's iterations do not converge, Differential equations of first order not solved for the derivative, Uniqueness of solutions with a given slope, Singular solutions, p- and c-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and nonexistence of singular solutions, examples.

UNIT TWO:

Systems of I order equations arising out of equations of higher order, Norm of Euclidean spaces convenient for analysis of systems of equations, Lipschitz condition for functions from $Rn+1$ to Rn, Local existence and uniqueness theorems for systems of I order equations, Gronwall's inequality, Global existence and uniqueness theorems for existence of unique solutions over whole of the given interval and over whole of R, Existence

theory for equations of higher order, Conditions for transformability of a system of I order equations into an equation of higher order.

UNIT THREE:

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

UNIT FOUR:

Convergence of real power series, Radius and interval of convergence, Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points.

UNIT FIVE:

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Orthogonality relations, Fourier-Legendre expansion, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Sturm comparison theorem, Zeros of Bessel functions, Orthogonality relations, Generating function, Fourier-Bessel expansions.

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.

MAT3TH07: FLUID MECHANICS

UNIT ONE: Review of basic concepts, Real and ideal fluids, Newton's law of viscosity, Convective transport of scalar and vector quantities, Differentiation following the motion and acceleration, The equation of Continuity, Velocity Potential , Body forces, surface forces, Stress vector at a point, Nature of stresses, Stress on an Arbitrary Plane: Cauchy's Stress formula, State of stress at a point, Stress tensor, Isotropic Law of Pressure, Principal stresses and Principal Directions, Stress invariants, General displacement of a fluid element.

UNIT TWO: Nature of strains, Rates of strain components, Relation between stress and rates of strain , Transformation of Stress- Components, Transformation of Rates of Strain, Navier-Stokes equation, Euler's Equation, Energy Dissipation due to Viscosity, Diffusion of Vorticity,

UNIT THREE: Stream tube and Vortex tube, Helmholtz's vorticity theorem, Kelvin's Circulation theorem, Energy Flux, Mean Potential over a spherical surface in a simply connected region, Kinetic Energy in Irrotational Flow, Kelvin's Minimum kinetic energy Theorem, Uniqueness of the Irrotational motion

UNIT FOUR: Two dimensional irrotational motion, The stream function, The Complex potential for two dimensional irrotational motion, Concept of line- sources, sinks, doublets and vortices, Superposition of solutions, The concept of Images, The Vortex pair, Vortex rows: Single infinite row of Line Vortices,the Karman vortex street, Milne-Thomson Circle Theorem, Blasius Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder, Conformal transformation: Uniform line distributions (source, vortex and doublet) under conformal transformation.

UNIT FIVE: Three dimensional irrotational flow, Concept of Sources, Sinks and Doublets, Axisymmetric flows, Stokes stream function, Statements of Weiss's and Butler's sphere theorems and their applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at rest at infinity,
Gravity waves – Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

Books Recommended:

1.L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
3.S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
4.A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
5. F. Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi, 1985

M.A./M.Sc. Semester –III Paper -5

MAT3TH51: RIEMANNIAN GEOMETRY

UNIT 1 : Riemannian metrics, Riemannian manifolds, examples, affine connections, covariant differentiation of tensor fields, covariant derivative along a curve, parallel transport, Levi-Civita connection, fundamental theorem of Riemannian geometry.

UNIT 2 : Differential operators on Riemannian manifolds, gradient vector fields, divergence of a vector field, Laplacian operator, Lie derivative of tensor field with respect to a vector field.

UNIT 3 : Riemannian curvature tensor, identities satisfied by Riemannian curvature tensor, sectional curvature, Schur's theorem, Ricci curvature, scalar curvature, Einstein manifolds, isometries, notion of covering spaces, pull-back metrics via diffeomorphisms.

UNIT 4 : Length of a curve, Riemannian distance function, geodesics, local existence and uniqueness of geodesics, exponential map, Gauss Lemma, minimizing roperties of geodesics, geodesic normal coordinates.

UNIT 5 : Jacobi fields, conjugate points, complete Riemannian manifolds, Hopf-Rinow theorem, The theorem of Hadamard, Riemannian submanifolds, second fundamental form, Gauss equation, Ricci equation, model spaces of constant curvature

Books Recommended:

1. M. P. do Carmo ; Riemannian Geometry, Berkhauser, 1992.
2. P. Peterson; Riemannian Geometry, Springer, 2006.
3. J. Jost; Riemannian Geometry and Geometrical Analysis, Springer, 6th edition, 2011.
4. J. M. Lee; Riemannian Manifolds: An Introduction to curvature, Springer, 1997.
5. S. Gallot, D. Hullin, J. Lafontaine; Riemannian Geometry, Springer, 3rd edition, 2004.

M.A./M.Sc. Semester –II Paper -1

MAT2TH02: MODULE THEORY

UNIT ONE: Modules over a ring, Endomorphism ring of an abelian group, R-Modulestructure on an abelian group M as a ring homomorphism from R to $\mathrm{End}_Z(M)$, submodules, Direct summands, Annihilators, Faithful modules, Homomorphism, Factor modules, Statements of Correspondence theorem and Isomorphism theorems, $\mathrm{Hom}_R(M, N)$ as an abelian group and $\mathrm{Hom}_R(M, M)$ as a ring, Exact sequences, Five lemma, External and internal direct sums and their universal property.

UNIT TWO: Free modules, Homomorphism extension property, equivalentcharacterization as a direct sum of copies of the underlying ring, existence of a basis of a vector space, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization, Divisible groups, Examples of injective modules.

UNIT THREE: Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Divisor chain condition, Unique factorization domains, Examples and counterexamples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Unique factorization in polynomial rings over UFD's.

UNIT FOUR: Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into $T(M)$ and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT FIVE: Reduction of matrices over polynomial rings over a field, Similarity of matrices and $F[x]$-module structure, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Diagonalizable and nilpotent parts of a linear operator, Jordon-Chevalley Theorem

Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. F. W. Anderson and K. R. Fuller, Rings and Categories of Modules, Springer, N.Y., 1974.
3. I. A. Adamson, An Introduction to Field Theory. Oliver & Boyd, Edinburgh, 1964.
4. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Ltd., New Delhi, 1986.

Further Reading:

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. P. Ribenboim, Rings and Modules, Wiley Interscience, N.Y., 1969.
3. J. Lambek, Lectures on Rings and Modules, Blaisedell, Waltham, 1966.
4. Ramji Lal, Algebra, Vols. II, Shail Publications, Allahabad, 2002.

DEPARTMENT OF MATHEMATICS

EWING CHRISTIAN COLLEGE, PRAYAGRAJ

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M.A./M.Sc. Semester –II Paper -2

MAT2TH04: MEASURE AND INTEGRATION

UNIT ONE: Countable and uncountable sets, cardinality and cardinal arithmetic, Schröder-Bernstein theorem, $a < 2^a, 2^{\aleph_0} = c$, the Cantor's ternary set and its properties.

UNIT TWO: Semi-algebras, algebras, monotone class, σ-algebras, measure and outermeasures, Caratheödory extension process of extending a measure on a semi-algebra togenerated σ -algebra, completion of a measure space.

UNIT THREE: Borel sets, Lebesgue outer measure and Lebesgue measure on R,translation invariance of Lebesgue measure, existence of a non-measurable set,characterizations of Lebesgue measurable sets, the Cantor-Lebesgue function.

UNIT FOUR: Measurable functions on a measure space and their properties, Borel andLebesgue measurable functions, simple functions and their integralson R, Littlewood's three principles (statement only), Lebesgue integral on R and its properties.

UNIT FIVE: Bounded convergence theorem, Fatou's lemma, Lebesgue monotoneconvergence theorem, Lebesgue dominated convergence theorem, Minkowski's andHölder's inequalities, Riesz-Fischer theorem (statement only).

Books Recommended:

H. L. Royden and P. M. Fitzpatrick, Real Analysis, (Fourth Edition), Prentice Hall, 2010.

Further Reading:

1. P. R. Halmos, Measure Theory, Grand Text Mathematics, 14, Springer, 1994.
2. E. Hewit and K. Stromberg, Real and Abstract Analysis, Springer, 1975.
3. K. R. Parthasarathy, Introduction to Probability and Measure, TRIM 33,Hindustan Book Agency, New Delhi, 2005.
4. I. K. Rana, An Introduction to Measure and Integration, (Second Edition), NarosaPublishing House, New Delhi, 2005.

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M.A./M.Sc. Semester –II Paper -3

MAT2TH06: PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Unit-I: Formation of P.D.E's, First order P.D.E.'s, Classification of first order P.D.E.'s, Complete, general and singular integrals, Lagrange's or quasi-linear equations, Integral surfaces through a given curve, Orthogonal surfaces to a given system of surfaces, Characteristic curves.

Unit-II: Pfaffian differential equations, Compatible systems, Charpit's method, Jacobi's Method.

Unit-III: Linear equations with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.'s.

Unit-IV: Method of separation of variables:- Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations of strings and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

Unit-V: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books recommended:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill Pub.,(1957)
2. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Pub. (2005)
3. R.P. Kanwal: Linear Integral Equations, Birkhauser Verlag Pub.(1997)

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M.A./M.Sc. Semester –II Paper -4

MAT2TH08: MATHEMATICAL METHODS

UNIT ONE:

Boundary-value problems: Orthogonal and Orthonormal sets of functions, Sturm-Liouville (S-L) problems, Eigenvalues and Eigenfunctions of S-L problems , Reality of Eigenvalues and Orthogonality of Eigenfunctions of S-L problems, Singular Sturm-Liouville problems, Mean square Convergence, Completeness of Orthonormal sets, Bessel's inequality, Self-adjoint differential equations. Orthogonal Eigenfunction Expansions, Generalized Fourier series.

UNIT TWO:

Fourier Series: Periodic functions, Trigonometric series, Fourier series, Euler formulas, A set of sufficient conditions for the convergence of Fourier series of a continuous function of period 2π, Functions of arbitrary periods, Even and Odd functions, Fourier Cosine and Sine series, Half-range expansions, Complex Fourierr series, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error, Bessel's inequality

UNIT THREE:

From Fourier Series to Fourier Integral, Sufficient conditions for the validity of Fourier integral representation, Fourier Cosine and Sine Integrals, Fourier Cosine and Sine Transforms, Linearity and Fourier Cosine and Sine Transforms of Derivatives, Complex form of Fourier Integral, Fourier Transform and its Inverse, Linearity , Shifting properties, Fourier Transform of Derivatives. Convolution

UNIT FOUR:

Definition, Linearity and Existence of Laplace Transform. The Inversion formula, Laplace Transform of the Derivatives and of the Integrals of a function, Unit step function Shifting Theorems, Derivatives and Integrals of Laplace Transforms, Convolution products , Application to the Initial Value Problems and System of ODE.

UNIT FIVE:

Calculus of Variations: Functionals and extremals, Variation and its properties, Euler equations, Cases of several dependent and independent variables, Functionals dependent on higher derivatives, Simple applications.

Books Recommended:

1.E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8thEdition, 2001.
2.A. D. Polyanin and A. V. Manzhirov, Handbook of Integral Equations, CRC Press, 2ndEdition, 2008.
3. L. Elsgolts, Differential Equations and Calculus of Variations, Mir Publishers, 1970.
4. A. S. Gupta, Calculus of Variations, Prentice Hall of India, New Delhi, 1999.
5. J. H. Davis, Methods of Applied Mathematics with a MATLAB Overview, Birkhäuser, Inc., Boston, MA, 2004
6. William E. Boyce and Richard C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley & Sons, (Asia), Seventh Edition, 2003.
7. Pipes,Applied Mathematics for Engineers and Physicists, McGraw-Hill International Student Edition, 2nd edition

M.A./M.Sc. Semester –II Paper -5

MAT2TH10: DIFFERENTIAL GEOMETRY II

Unit 1: n-dimensional real vector space, contravariant vectors, dual vector space, covariant vectors, tensor product, second order tensors, tensors of type (r, s), symmetry and skew symmetry of tensors, fundamental algebraic operations, quotient law of tensors.

Unit 2 : Topological manifolds, compatible charts, smooth manifolds, examples, smooth maps and diffeomorphisms, definition of a Lie group, examples.

Unit 3: Tangent and cotangent spaces to a manifold, derivative of a smooth map, immersions and submersions, submanifolds , vector fields, algebra of vector fields, φ-related vector fields, left and right invariant vector fields on Lie groups.

Unit 4: Integral curves of smooth vector fields, complete vector fields, flow of a vector field, distributions, tensor fields on manifolds, r-forms, exterior product, exterior differentiation, pull-back differential forms.

Unit 5: Affine connections (covariant differentiation) on a smooth manifold, torsion and curvature tensors of an affine connection, identities satisfied by curvature tensor.

Books Recommended:

1. Kobayashi and Nomizu; Foundations of Differential geometry, Vol-1, Interscience Publishers, 1963.
2. T. J. Willmore; Riemannian geometry, Oxford Science Publication, 1993.
3. S. Kumaresan; A course in Differential Geometry and Lie groups, Hindustan Book Agency, 2002.
4. M. Spivak; A comprehensive Introduction to Differential Geometry, Vols. 1-5, Publish or Perish, Inc., Houston, 1999.
5. W. M. Boothby; An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.
6. U. C. De, A. A. Sheikh; Differential Geometry of Manifolds, Narosa Publishing House, 2007.
7. R. S. Mishra, A course in Tensors with Applications to Riemannian Geometry, Pothishala, Pvt. Ltd., Allahabad, 1965.

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M.A./M.Sc. Semester –IV Paper -1

MAT4TH02: WAVELETS

Theory: 75%

UNIT ONE: The discrete Fourier transform and the inverse discrete Fourier transform, their basic properties and computations, The fast Fourier transform, The discrete cosine transform and the fast cosine transform.

UNIT TWO: Construction of wavelets on $\mathbb{Z}N$, First stage and by iteration, The Haar system, Shannon wavelets, Daubechies' D6 wavelets on $\mathbb{Z}N$.
Description of $l2(\mathbb{Z})$, $L2[-\pi, \pi]$, $L2(\mathbb{R})$, their orthonormal bases, Fourier transform and convolution on $l2(\mathbb{Z})$, wavelets on Z, Haar wavelets on Z, Daubechies' D6 wavelets for $l2(\mathbb{Z})$.

UNIT THREE: Orthonormal bases generated by a single function in $L2(\mathbb{R})$, Fourier transform and inverse Fourier transform of a function f in $L1(\mathbb{R})$ I $L2(\mathbb{R})$, Parseval's relation, Plancherel's formula, Orthonormal wavelets in $L2(\mathbb{R})$, Balian-Low theorem.

UNIT FOUR: Multi-resolution analysis and MRA wavelets, certain function in $L2(\mathbb{R})$ for which $\{\psi j, k\}$ does not form an orthonormal system, compactly supported wavelets, band-limited wavelets.

UNIT FIVE: Franklin wavelets on R, Dimension function, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journe's wavelet. Decomposition and reconstruction algorithms of Wavelets.

Lab-work: 25%
The following lab work is recommended:
1. To plot a member in $l2(\mathbb{Z}N)$, its Fourier transform and its inverse Fourier transform.
2. To verify various identities relating Fourier transform, inverse Fourier transform, inner product, norm and convolution.
3. Computing Fourier coefficients of an element of $l2(\mathbb{Z}N)$,with respect to a given wavelet (Haar, Daubechies' D6) at a certain level such as $z, \psi-2, k$ etc.

Books Recommended:

1. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer-Verlag, 1999.
2. Eugenio Hernández and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.

Further Reading:

1. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
2. Ingrid Daubechies, Ten Lectures on Wavelets, CBS-NFS Regional Conferences.

DEPARTMENT OF MATHEMATICS

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M.A./M.Sc. Semester –IV Paper -2

MAT4TH54: ADVANCED MODULE THEORY

UNIT 1 Modules over rings, modular law, annihilators, factor theorem, projections, idempotent endomorphisms.

UNIT 2: Chain conditions on modules, Noetherian modules and rings, artinian modules and rings, equivalent characterizations, composition series of modules, Jordan–Hölder theorem and Hilbert basis theorem (statement only).

UNIT 3: Fitting lemma**,**Jacobson radical, Jacobson semi-simple ring, nilpotent and nil ideals, Hopkins Levitzki theorem, Nakayama's Lemma.

UNIT 4: Injective modules and divisible modules, embedding theorem for modules, essential extension, injective envelope of a module.

UNIT 5: Small submodules, projective modules and projective covers, Jacobson radical of a projective module.

References:

1. F. W. Anderson and K. R. Fuller, Rings and Category of Modules, Graduate texts in mathematics, Vol 13, Springer-Verlag Inc., New York, 1974.
2. Paul E. Bland, Rings and their Modules, Walter de Gruyter GmbH & Co. KG, Berlin/New York, 2011.
3. T. Y. Lam, A First Course in Noncommutative Rings, Graduate texts in mathematics: 131, Springer-Verlag New York, Inc., 1991.
4. D. W. Sharpe and P. Vamos, Injective Modules, Cambridge University Press, 1972.

Further Readings:

1. O. Zariski and P. Samuel, Commutative Algebra, Volume I, D. Van Nostrand Company, Inc., 1958.
2. T. Y. Lam, Lectures on Modules and Rings,Graduate texts in mathematics: 189, Springer-Verlag New York, Inc., 1999.
3. D. S.Dummit and R. M. Foote, Abstract Algebra, Third Edition, John Willy and Sons, Inc., 2004.

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M.A./M.Sc. Semester –IV Paper -3

MAT4TH72 : LIE ALGEBRAS

UNIT ONE: Definition and examples of Lie algebras, classical Lie algebras, derivations of a Lie algebra, abelian Lie algebra, Lie subalgebras, ideals and homomorphisms, normalizers and centralizers of a Lie subalgebra, representations of lie algebras (definition and some examples), automorphisms of a Lie algebra.

UNIT TWO: Solvable algebra, Solvable radical, nilpotent algebra, Engel's Theorem, semi-simple Lie algebra, Lie's Theorem, Jordan-Chevalley decomposition (existence and uniqueness), Cartan trace criterion for solvability, Killing form and criterion for semisimplicity.

UNIT THREE: Simple ideals, inner derivations, abstract Jordan-Chevalley decomposition, Lie algebra modules, Schur's Lemma, Casimir elements of a representation, Weyl's Theorem for preservation of Jordan-decomposition.

UNIT FOUR: Representation of sl(2, **C**): weights, highest weight, maximal vectors, classification of irreducible modules, toral and maximal toral subalgebra, root space decomposition and properties of roots.

UNIT FIVE: Abstract root systems, Weyl group, root strings, bases and their existence, Weyl chambers, Classification of rank 2 root systems.

Books Recommended:

1. J, E. Humphreys, Introduction to Lie Algebras and Representation Theory, Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, Lie Algebras, Wiley-Interscience, New York, 1962.
3. J. P. Serre, Lie Algebras and Lie Groups, Benjamin, New York, 1965.

M.A./M.Sc. Semester –IV Paper -4

MAT4TH74: Magnetohydrodynamics

Unit 1.

Maxwell's equations, Conservation of energy, Poynting vector, Conservation of momentum and Maxwell's stress tensor, Electromagnetic momentum density.

Unit 2.

Nature of Magnetohydrodynamics, Main assumptions of MHD, Electromagnetic fields in a conductor at rest, a uniformly moving rigid conductor and a deformable conductor. Basic equations of non- viscous and viscous magnetohydrodynamics : mass, momentum and energy conservation laws

Unit 3.

Basic Properties of the magnetic field and MHD terms: Magnetic Reynolds number, magnetic viscosity, magnetic pressure, magnetic diffusion and frozen- in- effect. Magnetohydrodynamic boundary conditions

UNIT 4.

Magnetohydrodynamic Flows, Formulation and solution of Linear flow, Flow between parallel plates Hartmann flow, Couette flow

Unit 5.

Magnetohydrodynamic Waves, Linearized equations , MHD waves in a perfectly conducting fluid, Alfven waves and magnetosonic waves

Books Recommended:

1. J.D.Jackson, Classical Electrodynamics, Wile Eastwern Limited, New Delhi, 1990
2. .L. D. Landau and E. M. Lifshitz, Classical Electrodynamics, Butterworth-Heinemann, 2nd Edition, 1984.
3. A. Jaffery, Magnetohydrodynamics, Oliver and Boyd, N.Y. 1966

M.A./M.Sc. Semester –IV Paper -5

MAT4TH86: STABILITY THEORY OF DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

UNIT ONE: Uncoupled and coupled linear Systems, Reduction of coupled linear system to uncoupled linear system, Exponentials of operators, Fundamental theorem for linear systems, Non-homogeneous linear systems.

UNIT TWO: Non-linear Autonomous system, Linearization, The phase plane & its phenomena, Critical points, Types of critical points, Phase plane analysis, Conservative systems.

UNIT THREE: Variational matrix, Stability analysis of linear and nonlinear systems using variational matrix, Liapunov Function, Stability by Liapunov's Direct Method.

UNIT FOUR: Mathematical model, Formulation of mathematical models, Classification of mathematical models, Malthusian growth model, Logistic growth model, Regrowth Model, Delayed differential models.

UNIT FIVE: Lotka-Volterra predation model, Rosenzweig-MacArthur model, Lotka-Volterra competition model, Lotka-Volterra models of mutualism, obligate and non-obligate mutualism, effect of mutualism on predator-prey and competitive systems

Books Recommended

1. Lawrence Perko, 2001, Differential Equations and Dynamical Systems, Springer-Verlag, New York, Inc.
2. G. F. Simmons, 1991, Differential Equations with Applications and Historical Notes, Tata-McGraw Hill.
3. H. I. Freedman, 1980, Deterministic Mathematical Models in Population Ecology, Marcel Dekker, New York.